

Basic Thrust and Pressure Equations for Black Powder Assuming: $\gamma = 1.35$ and $\varepsilon = 2.0$
 where γ is ratio of specific heats and ε is nozzle expansion ratio

One basic thrust equation is: $F = C_f P_c A_t$
 where F = Thrust (pounds)
 C_f = Thrust Coefficient (dimensionless)
 P_c = Chamber Pressure (psia)
 and A_t = Nozzle Throat Area (square inches)

The thrust coefficient may be determined if the nozzle's expansion ratio, atmospheric pressure (in psia), and the thermodynamic properties of the gases involved are known. An expansion ratio of 2.0 is optimum for a pressure ratio of about 10.011 (chamber pressure / atmospheric pressure). For these conditions, a nozzle used in a perfect vacuum will have a vacuum thrust coefficient ($C_{f_{vac}}$) of 1.4591.

The thrust coefficient at any chamber pressure may be calculated if we know the nozzle/gas combination's vacuum thrust coefficient, atmospheric pressure, and the nozzle's expansion ratio:

$$C_f = C_{f_{vac}} - (\varepsilon P_a / P_c)$$

Multiplying both sides of this equation by P_c gives:

$$C_f P_c = C_{f_{vac}} P_c - \varepsilon P_a$$

We can rearrange this to give

$$C_{f_{vac}} P_c = C_f P_c + \varepsilon P_a$$

If we now divide both sides of this equation by $C_{f_{vac}}$ we have:

$$P_c = (C_f P_c + \varepsilon P_a) / C_{f_{vac}}$$

or

$$P_c = 0.6854 C_f P_c + 16.5856$$

Since $C_f P_c$ may be determined by rearranging the thrust equation to give

$$C_f P_c = F / A_t$$

We can now obtain the chamber pressure of any point on a thrust curve if we know the throat size of our nozzle.

To determine the conditions necessary to produce this pressure, we can use the relationship:

$$P_c = (K n \rho C^* r) / g$$

Where Kn is the burn area ratio (Burn Area/Nozzle Throat Area), ρ is the propellant density (in the neighborhood of 0.068 lbs/in³ for Black Powder), C^* is the Characteristic Exhaust velocity (about 2200-2400 ft/s for Black Powder for most model rocket pressures – we often use about 2300 for most of our calculations), r is the burning rate of the powder at that pressure (in/s), and g is the gravitational constant (we use a value of 32.17 ft/s²).

For these conditions:

$$P_c = 4.8617 Kn r$$

or

$$r = (0.2057 P_c) / Kn$$

So, if we have propellant characterization information, we can use the above equations or modifications of them to design rocket motors. These designs have to be verified by testing and then modified as necessary to achieve desired performance, but they do put us in the ballpark and give us a means of comparing various designs with a minimum of testing equipment. The missing elements for calculating the general case and modifying for other propellants may be found in various standard texts. Some of the most available are Sutton, Hill and Peterson and Humble, Larson, et al., but there are almost unlimited other texts.

In addition to density, specific heat ratios, actual mechanical properties etc. we are concerned about the interrelationships of burn rate, pressure, and burn area ratios of our propellant. The burn rate can usually be described by the equation:

$$r = a(P_c)^n$$

$$\text{Kn by:}$$

$$\text{Kn} = b(P_c)^{(1-n)}$$

$$\text{and } P_c \text{ by:}$$

$$P_c = c(\text{Kn})^{1/(1-n)}$$

It is obvious that they are related, but in practice the exponents and coefficients will vary in our experimental determinations. As a rule of thumb, the relationships of our exponents should be close to theoretical if experimental methods are acceptable, and decent design work can be done, even if we only have good burn rate versus pressure data. There is at least one commercial grain design program (GDP PRO and GDP Lite sold by AED Electronics) that uses only burn rate data along with physical measurements of the grain(s).

We can produce almost anything on paper. We can design a “good” motor using high burn rate exponents, but we normally cannot produce a reliable motor with repeatable performance with high burn rate exponents. Small changes in dimensions change performance too much. We are not limited by physical strength in our theoretical work, but we are when we actually produce these motors. Ambient pressure and temperature affect performance. If motor itself is at a different temperature, performance will be affected. Use at different locations and temperatures will produce different pressures, thrust levels and burn times (of both delay and thrust) than we obtain in our testing. If the strength limits of materials used in their construction (including the propellant) are exceeded, they will fail. Variation in the quality, strength, and performance of the materials and methods used will produce variation in our performance and can lead to failures. If we deviate too far from parameters established by design and testing, we can have failures.

This is just a brief basic preliminary introduction, and is limited, but hopefully gives a start in the discussed areas.

Recurring problems that plague us are the variation in burning rates from one lot of powder to another and the decrease in burning rates over the years. We have to hope that rates have stabilized at current levels although we will have to wait to see if history fulfills that hope or not.

The drop in burning rates results in a drop in chamber pressure, which produces a drop in thrust levels and efficiency of the powder. This means that we end up using more powder for the same impulse and that the motor must burn longer to produce the same total impulse. We are often tempted to try to regain this performance by increasing pressures. This is more difficult than it appears to be. Quite often the increase in pressure in a given casing gives little or no increase in thrust level. This is because we attempt to raise pressure by reducing throat size while holding the casing ID constant. Since the thrust coefficient rises rapidly with increasing pressure at low pressures, we can sometimes increase thrust levels appreciably in this operating region. However, at higher pressures, thrust coefficient does not rise much with increases in pressure and the increase in pressure is offset by the decrease in nozzle throat area.

The obvious and simple answer is to increase casing ID. However, due to “installed” base of rocket kits and established casing OD limitations we are restricted as to just how much we can increase ID’s and pressures before we encounter casing failures due to increased pressure and decreased strength of the casing.

Another option might be to begin phasing out current kits and motor casing sizes in order to be able to use the increase in ID more effectively. Once again easier said than done, but probably a good idea to keep in mind.